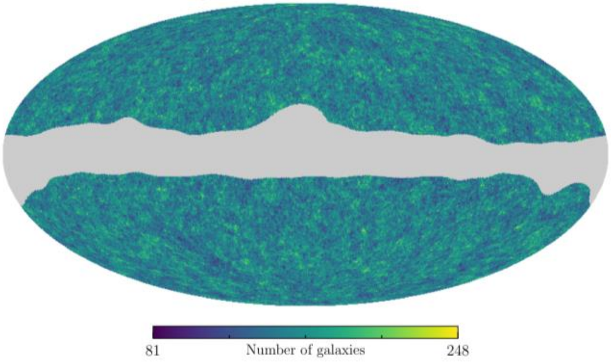
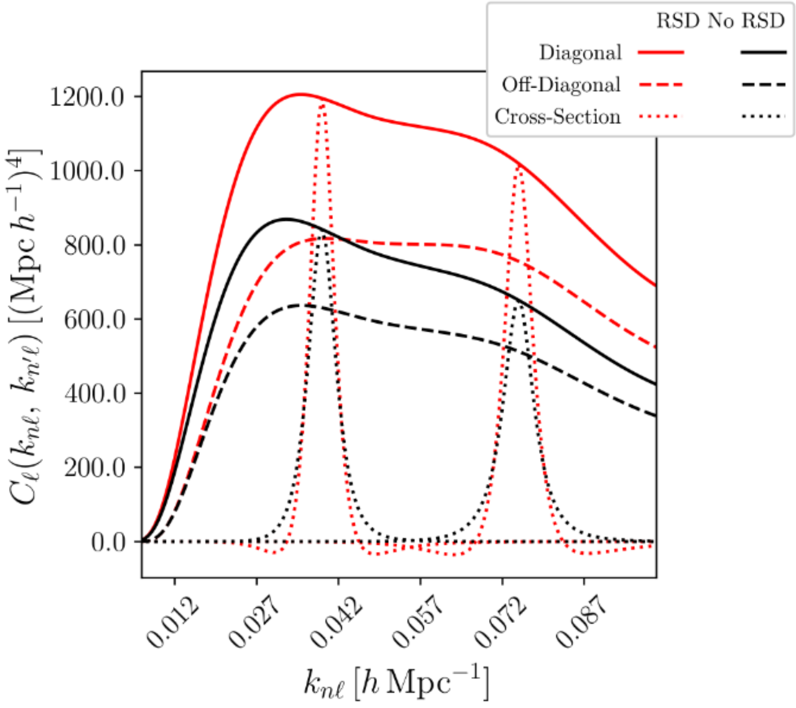




SuperFaB: a fabulous code for Spherical Fourier-Bessel decomposition

Henry S. Grasshorn Gebhardt

with Olivier Doré

NASA Postdoctoral Program Fellow

Jet Propulsion Laboratory
California Institute of Technology

Outline

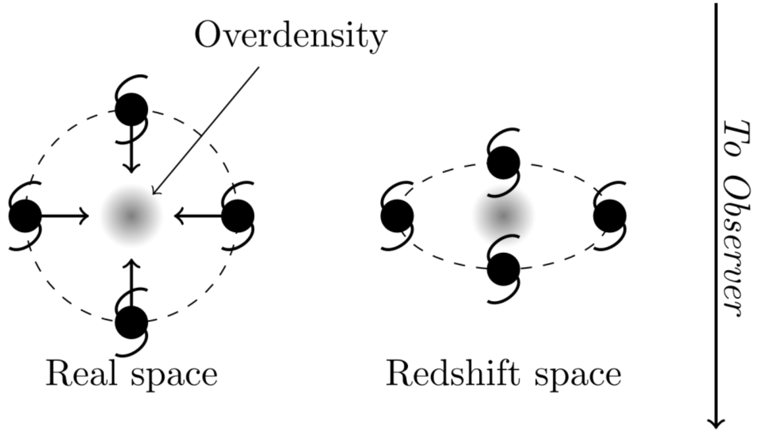
- Motivation: Cosmology with the Power Spectrum
  - RSD
  - Redshift-evolution
- Motivation: Spherical Fourier-Bessel (SFB) vs. Fourier
- Intuition for the SFB power spectrum
- SuperFaB, an SFB power spectrum estimator
- Testing, testing: *Roman*-, *SPHEREx*-, *Euclid*-like

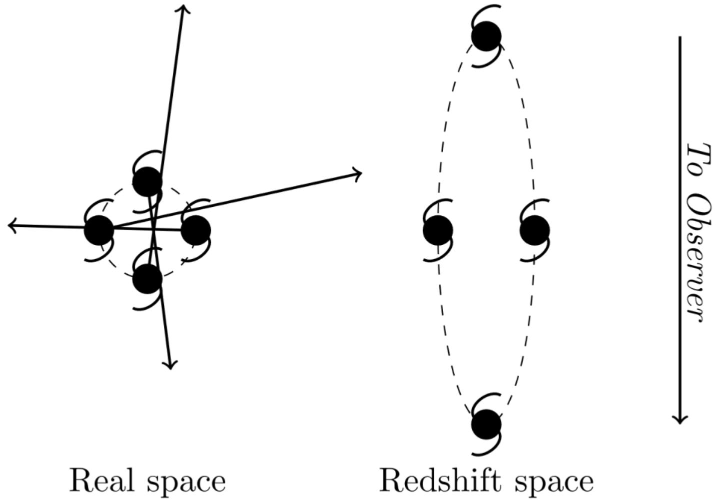
Paper: arXiv:2102.10079
Julia code: SphericalFourierBesselDecompositions.jl

Motivation: Roman, SPHEREx, Euclid, PFS, DESI, ...

- Galaxy surveys bridge the gap between low-redshift supernova Ia and high-redshift CMB measurements. (e.g. BAO)
- Redshift-space distortions (RSD) probe the growth of structure over cosmic time. → Tests for modified gravity.
- Large scales especially important for non-Gaussianity.
  - Are we seeing fNL?
  - Are we seeing relativistic effects?
  - Are we seeing wide-angle effects?

Redshift Space Distortions





Kaiser Effect:

Coherent flow of galaxies towards overdensities leads to increased clustering along the line of sight direction.

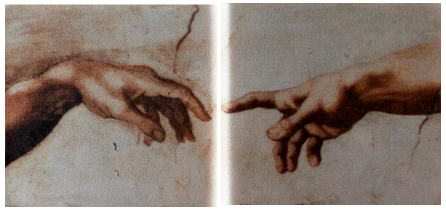
→ Non-zero quadrupole!

“Pancakes of God”

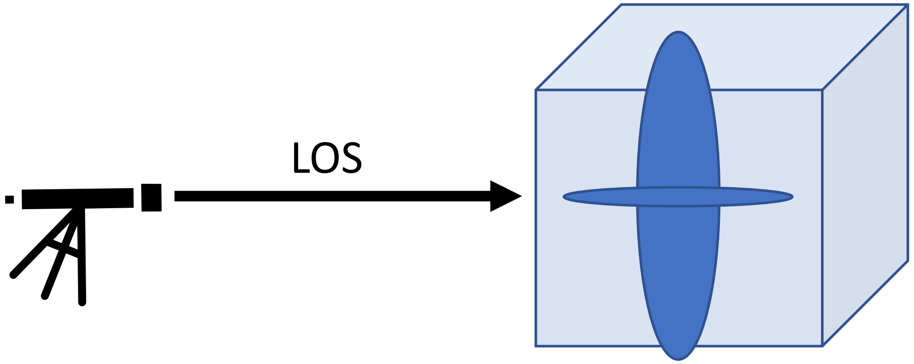
Fingers of God:

Stochastic motions of galaxies lead to a suppression of clustering along line-of-sight direction.

→ Small-scale suppression of power.



Homogeneity and ~~Isotropy~~ with Redshift-space Distortions (RSD)



$$\delta(\vec{r}) = \frac{n(\vec{r}) - \bar{n}}{\bar{n}}$$

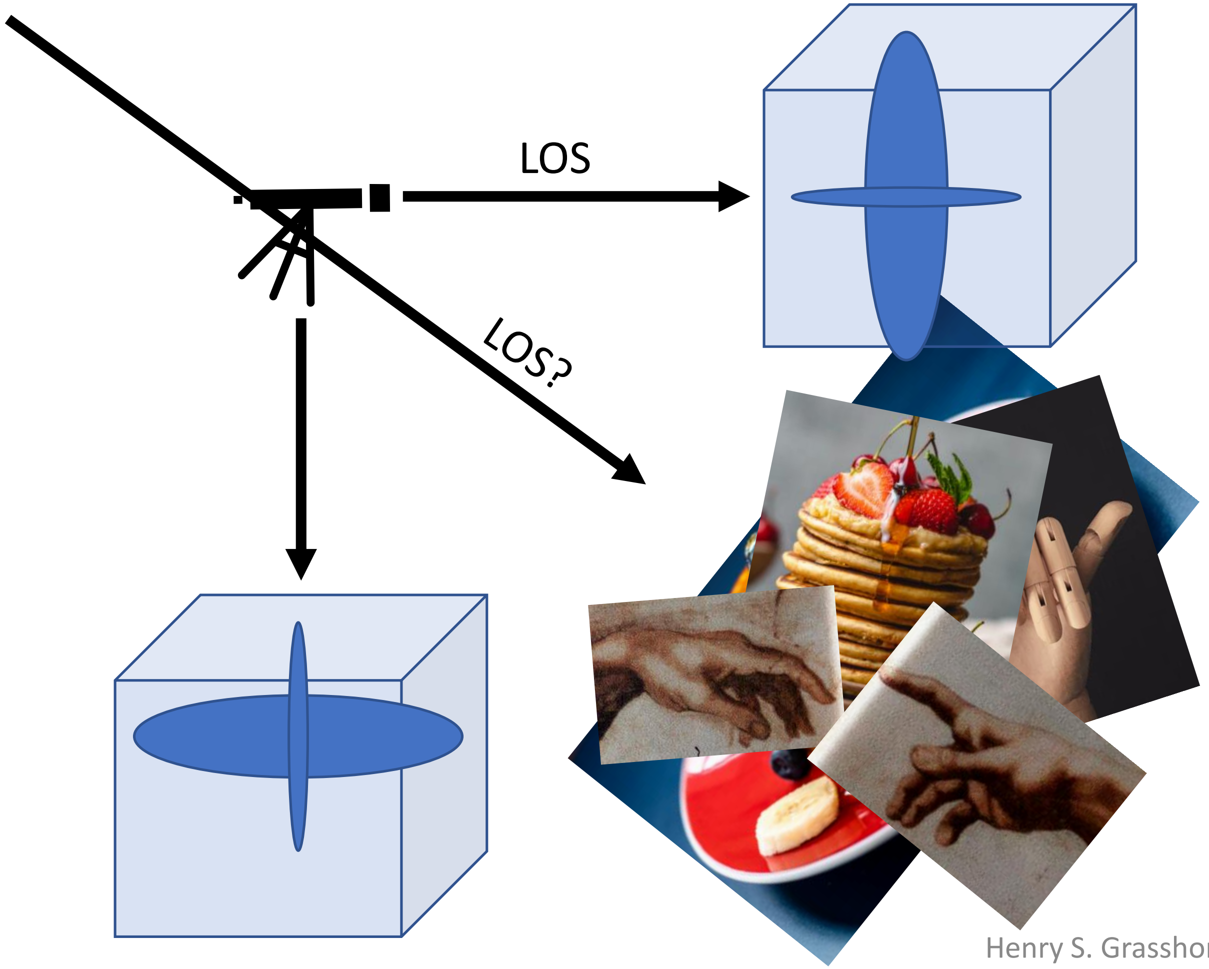
$$\delta(\vec{k}) = \int d^3r \, e^{-i\vec{k}\cdot\vec{r}} \, \delta(\vec{r})$$

$$\langle \delta(\vec{k}) \delta^*(\vec{k}') \rangle = (2\pi)^3 \delta^D(\vec{k}' - \vec{k}) P(k)$$

Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2 \, P(k)$

~~Homogeneity~~ and ~~Isotropy~~ with Redshift-space Distortions (RSD)



$$\delta(\vec{r}) = \frac{n(\vec{r}) - \bar{n}}{\bar{n}}$$

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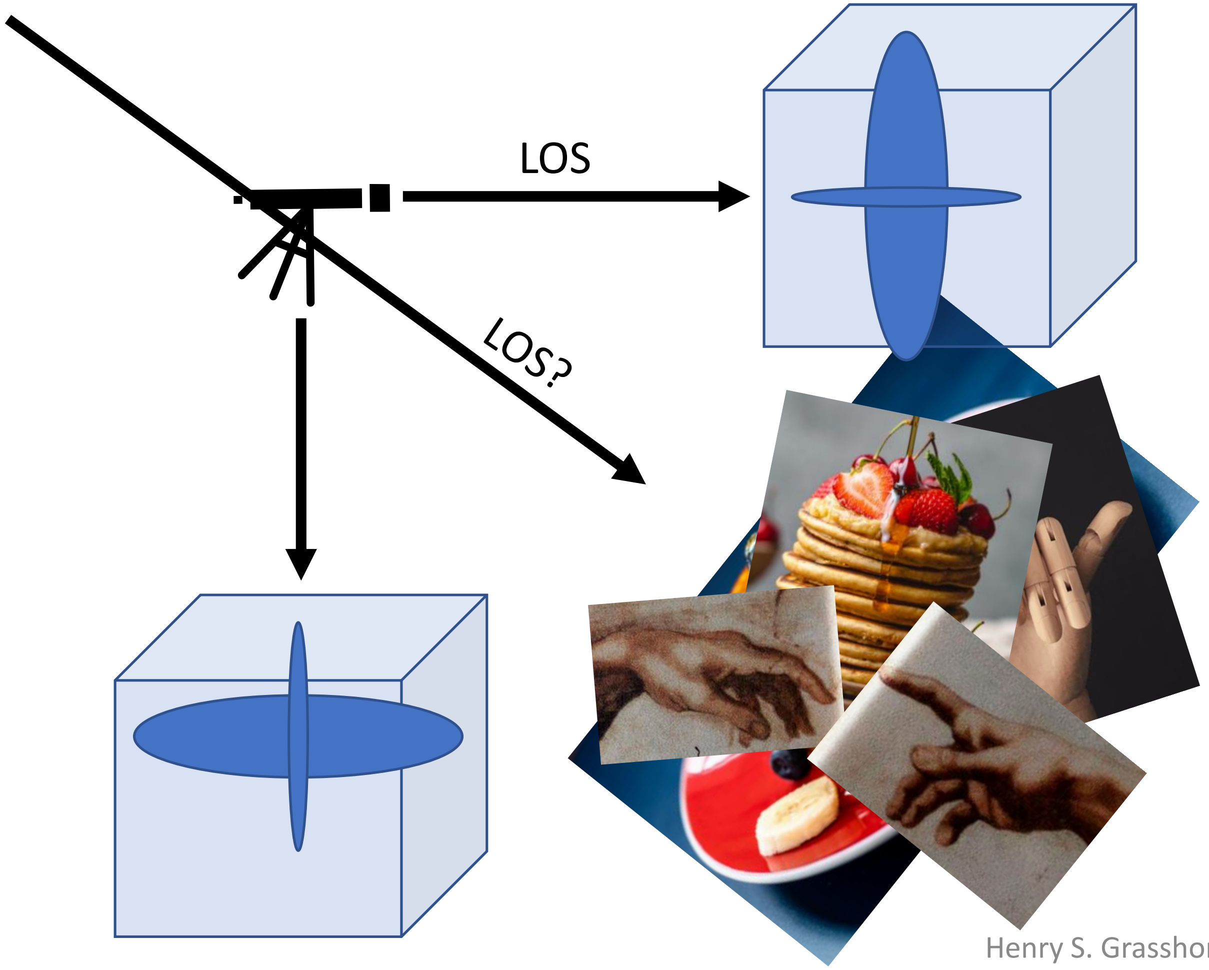
Homogeneity   Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2\, P(k)$

Nope: Wide angles

~~Homogeneity~~ and ~~Isotropy~~ with Redshift-space Distortions (RSD)



$$\delta(\vec{r}) = \frac{n(\vec{r}) - \bar{n}}{\bar{n}}$$

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$$\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle = (2\pi)^3 \delta^D(\vec{k}' - \vec{k}) P(k)$$

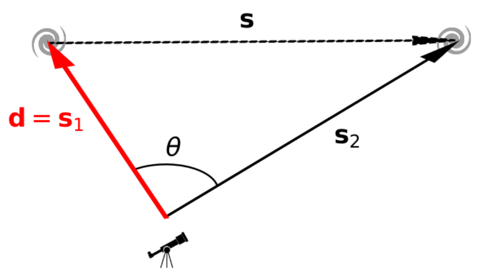
Homogeneity Isotropy

Nope: $P(k_\perp, k_\parallel) = (b + f\mu^2)^2\, P(k)$

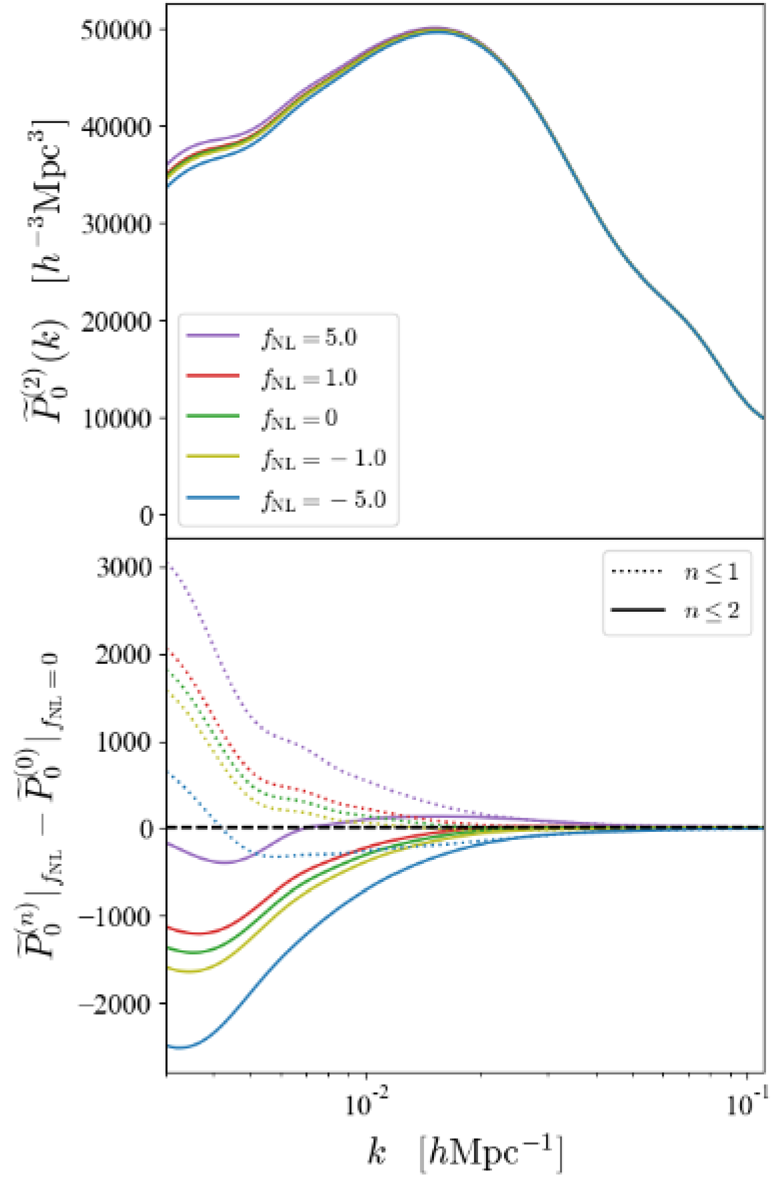
Nope: Wide angles

Yamamoto to the rescue: Great for small angles!

- Choose one LOS per galaxy pair.
- Perturbative expansion for modeling.
- Wide-angle effects can mimic non-Gaussianity.

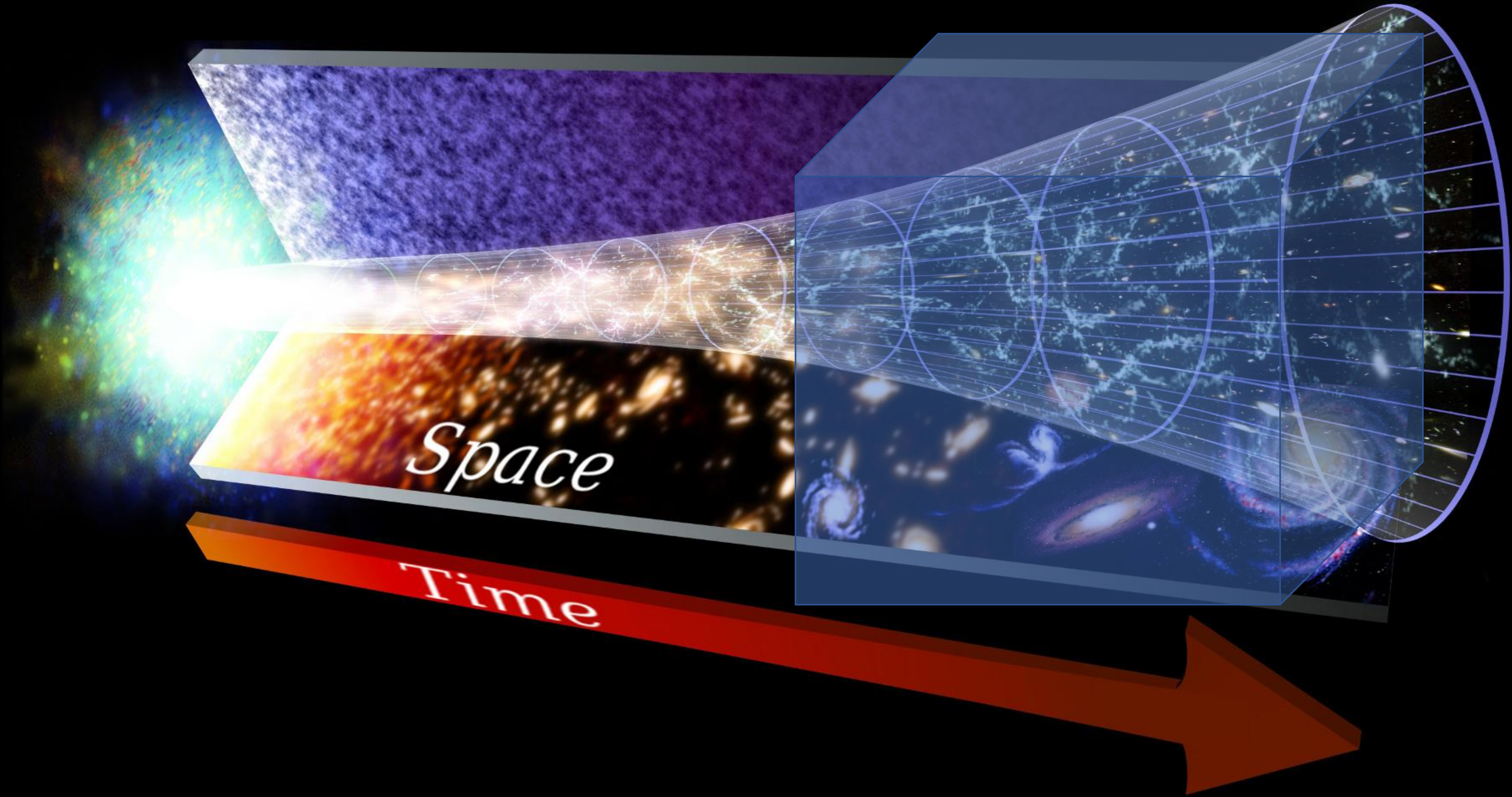


(Figure from Beutler, Castorina, Zhang 2019)



(Benabou, Sands, HGG + in prep)

Redshift-evolution: Our past lightcone is not homogeneous.



What to do?

Choose a coordinate system that is better suited for the fixed position of the observer than a Cartesian.

Laplacian eigenfunctions: $\nabla^2 f(\vec{r}) = -k^2 f(\vec{r})$

Cartesian Coordinates:

$$f(\vec{r}) = e^{-i\vec{k}\cdot\vec{r}}$$

Fourier transform:

$$\delta(\vec{k}) = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\, \delta(\vec{r})$$

Spherical Coordinates:

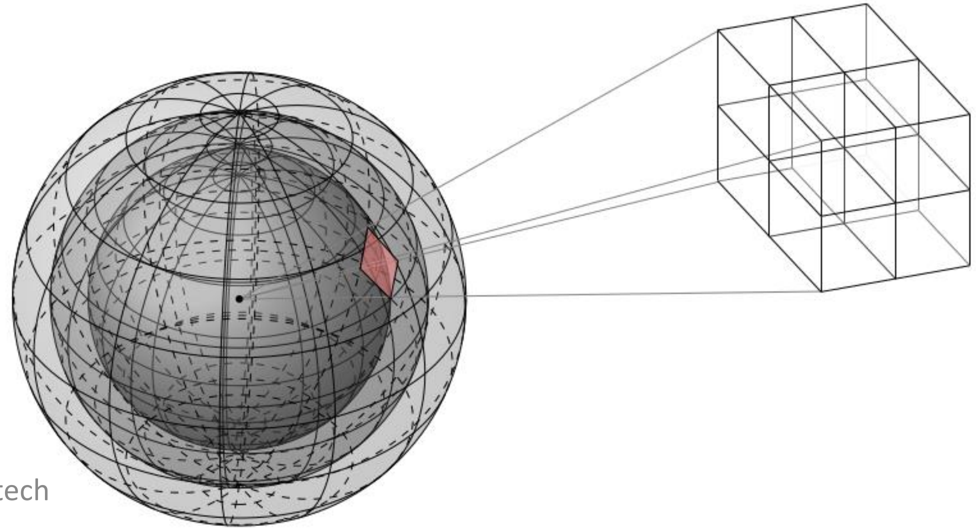
$$f(\vec{r}) = j_\ell(kr)\, Y_{\ell m}(\hat{r})$$

Spherical Fourier-Bessel transform:

$$\delta_{\ell m}(k) = \int d^3r\, j_\ell(kr)\, Y^*_{\ell m}(\hat{r})\, \delta(\vec{r})$$

The SFB power spectrum is ideally suited to deep and wide galaxy surveys

- Natural separation between **angular** and **radial** coordinates
  - Individual line of sights for each galaxy
    - Maximal information from RSD
  - All wide-angle effects
  - Redshift-evolution (e.g., growth of the non-linear power spectrum)
- Nearly diagonal covariance matrix



SFB power spectrum

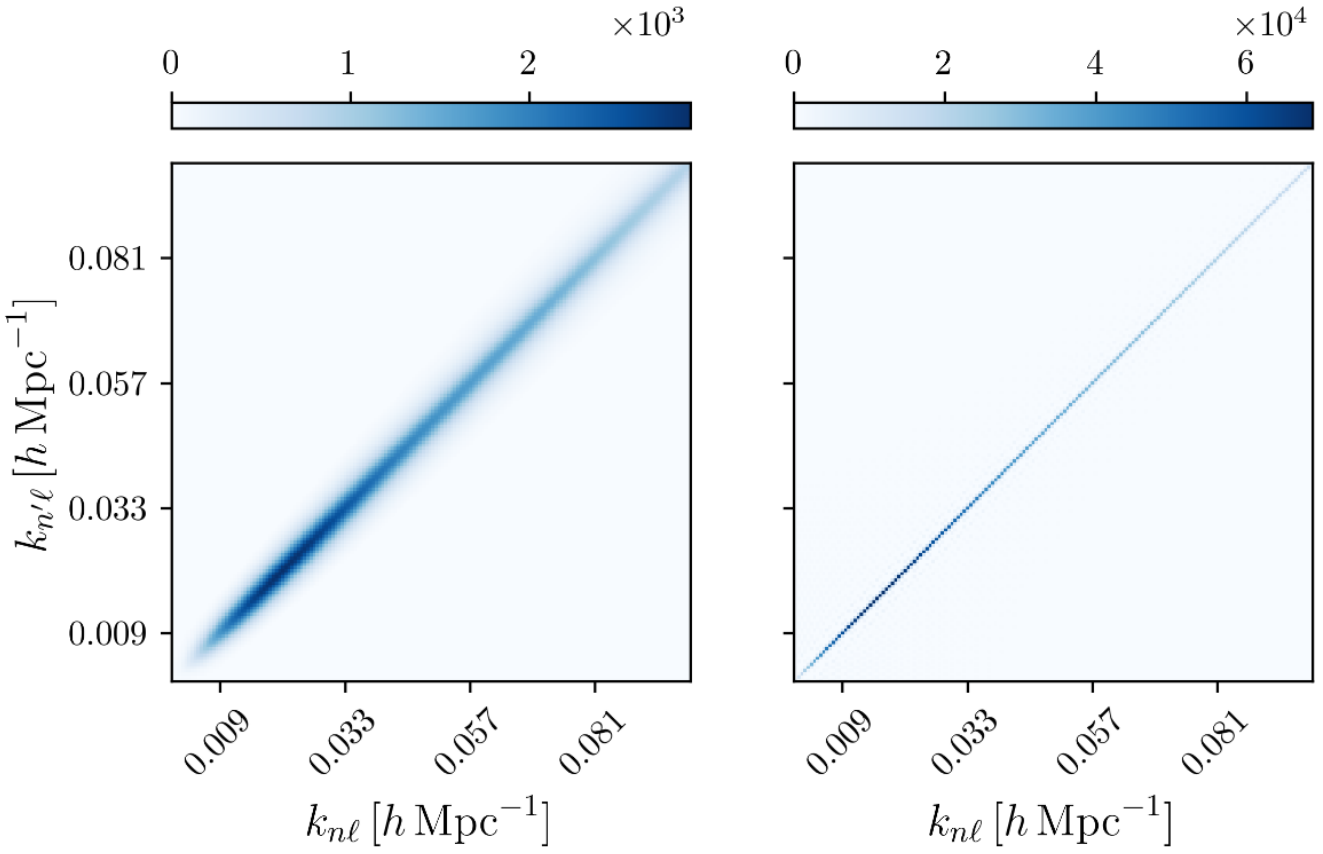
$$\langle \delta_{\ell m}(k)\, \delta_{\ell' m'}(k') \rangle = \underbrace{\delta^K_{\ell\ell'} \delta^K_{mm'}}_{\text{Isotropy}} C_\ell(k, k')$$

Isotropy

$$C_\ell(k, k') = \underbrace{\delta^D(k' - k)}_{\text{Homogeneity}} P(k)$$

Homogeneity

$\ell = 2$

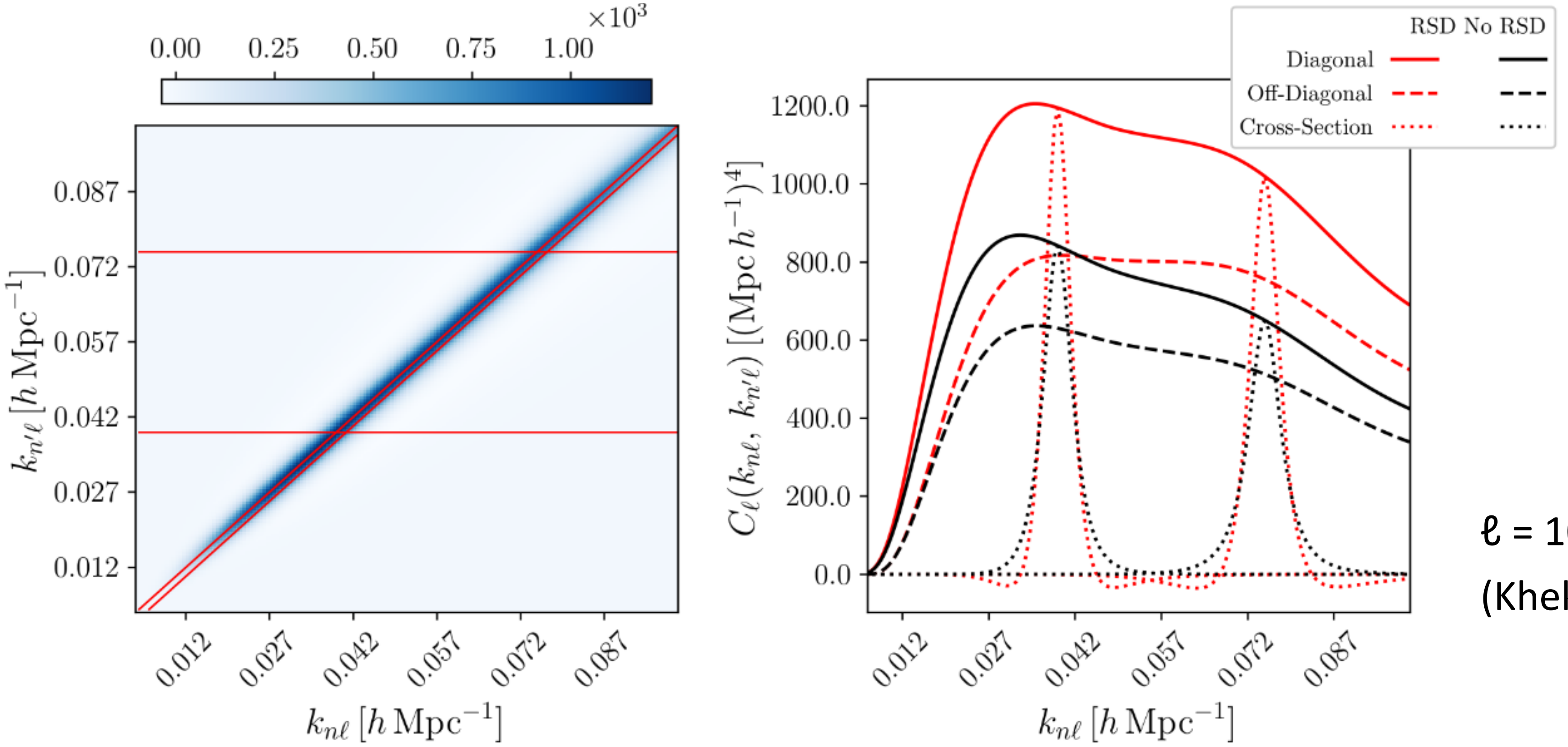


With redshift evolution

No redshift evolution

(Khek, HGG + in prep)

Redshift Space Distortions in SFB space



ℓ = 10

(Khek, HGG, + in prep)

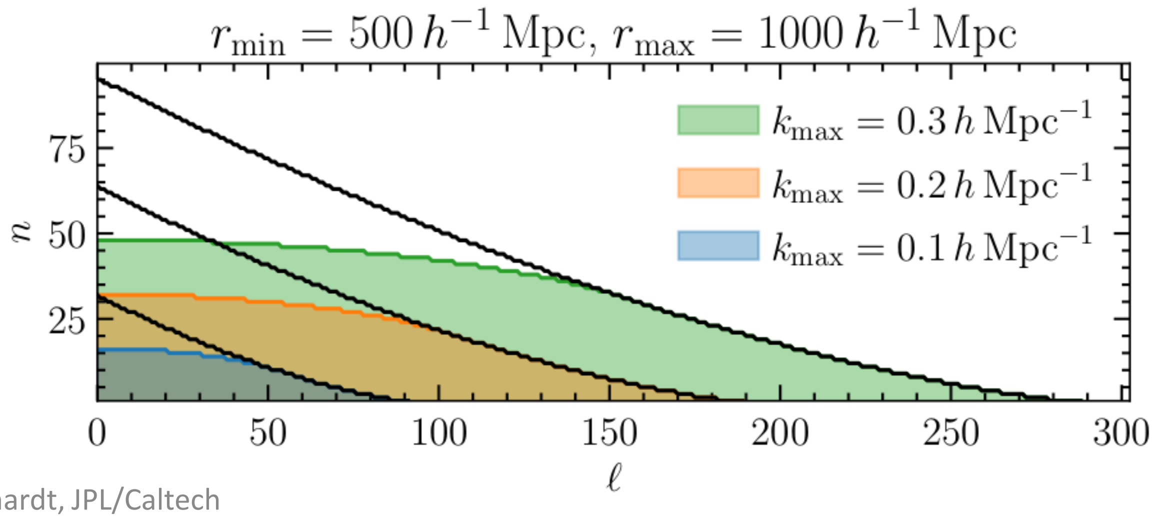
Linear Kaiser effect (pancakes!) add power on large scales.

Basic Algorithm for the Estimator (*SuperFaB*)

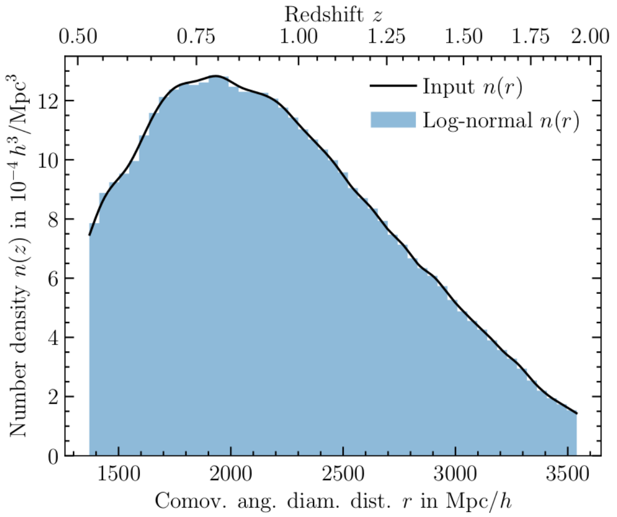
1. Radial Bessel transform with individual galaxies (Leistedt *et al* 2012)
2. Spherical harmonic transform with HEALPix (Gorski *et al* 2005)
3. Pseudo-SFB power spectrum bandpowers with window corrections (Hivon *et al* 2002)

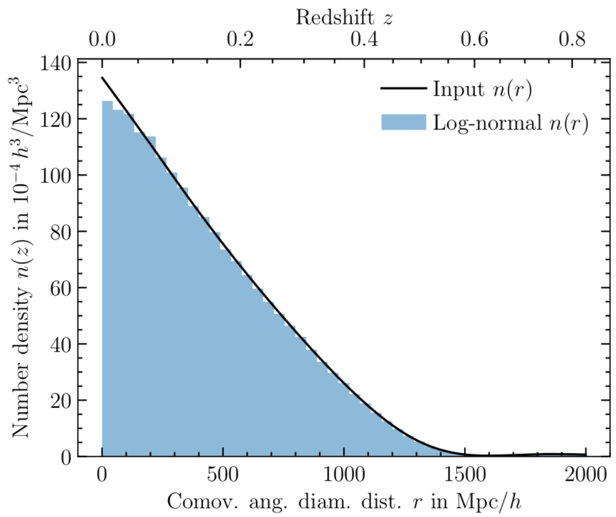
$$\hat{C}^{\rm obs}_{\ell n n'} = \frac{1}{2\ell+1} \sum_m \delta^{\rm obs}_{n\ell m} \delta^{\rm obs,*}_{n'\ell m}$$

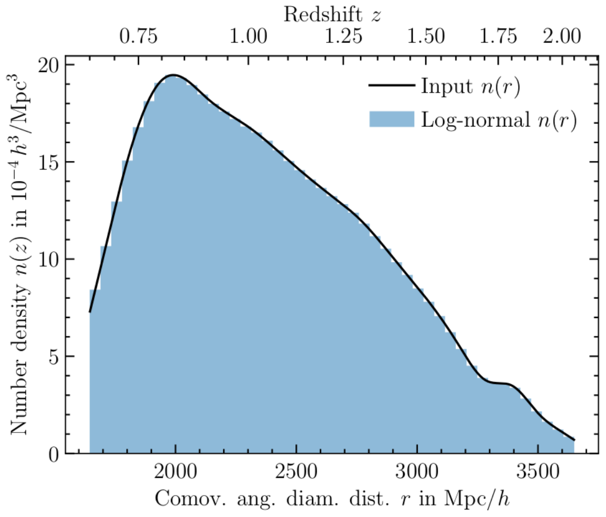
4. Subtract shot noise

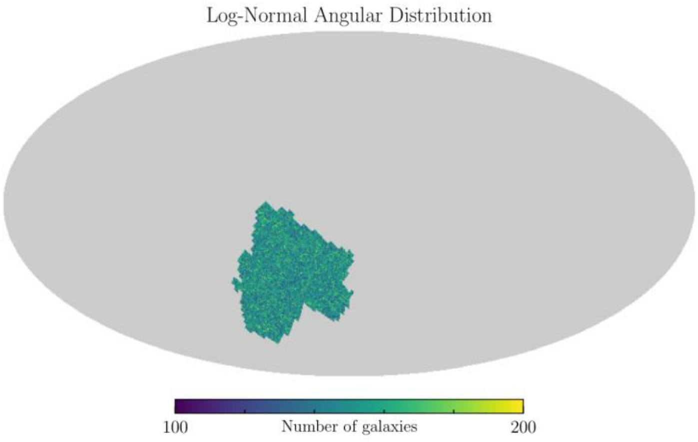


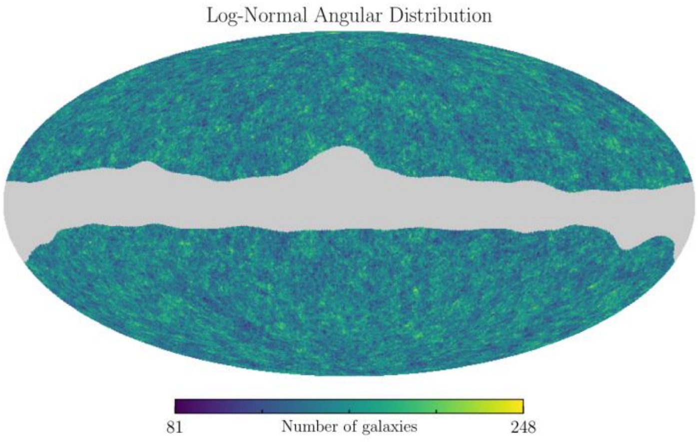
Examples: *Roman*-, *SPHEREx*-, *Euclid*-like

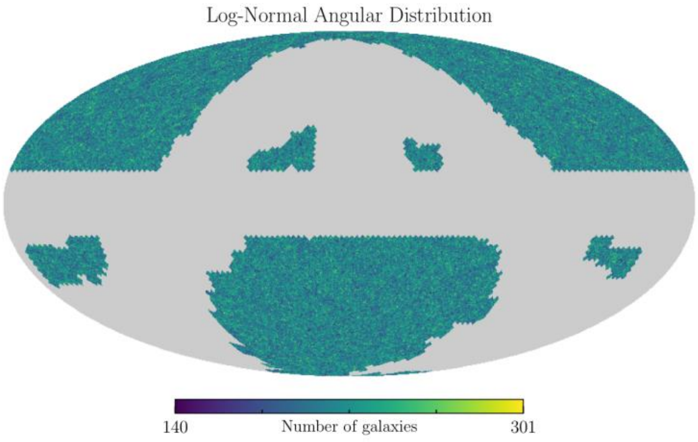




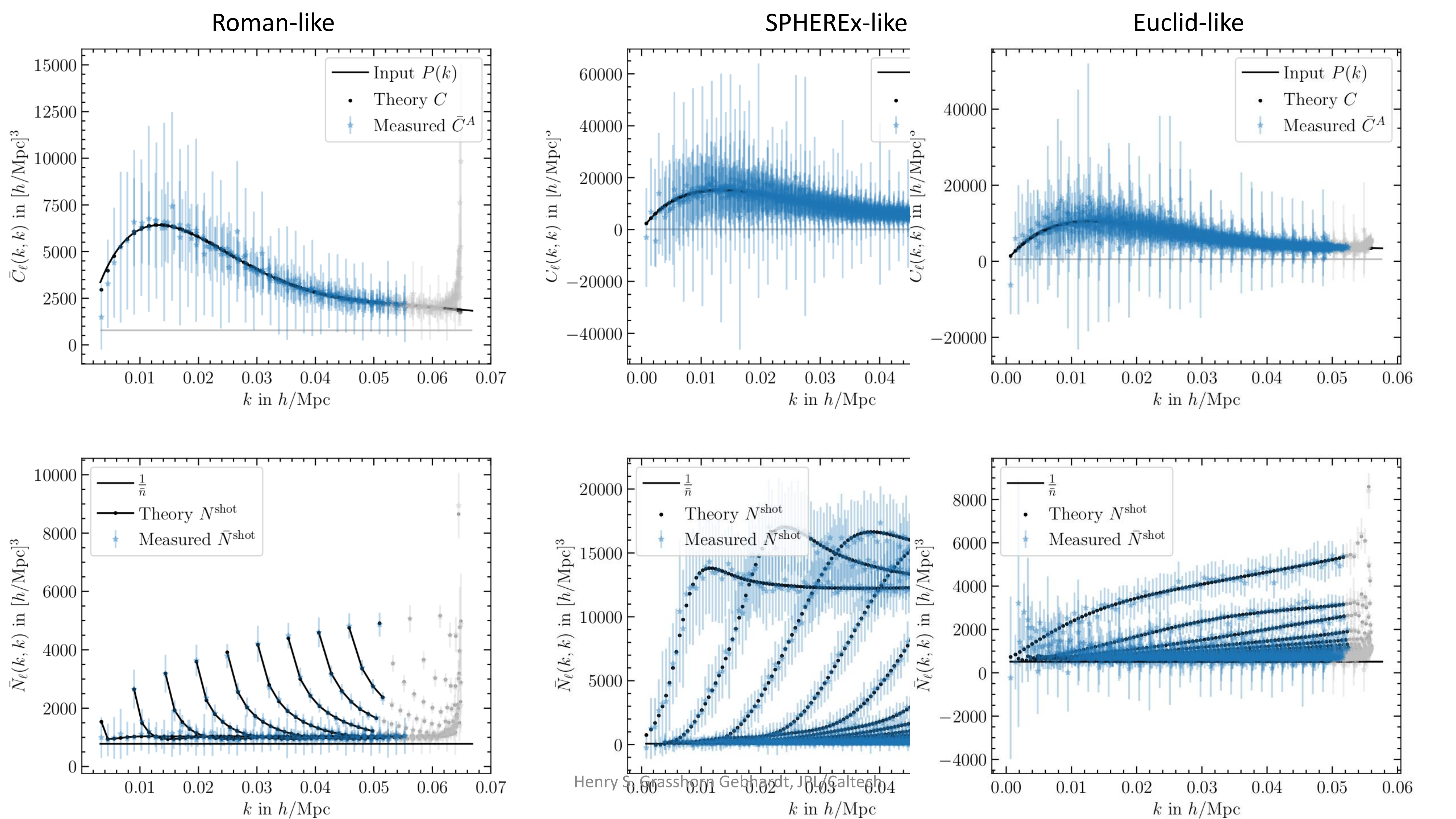








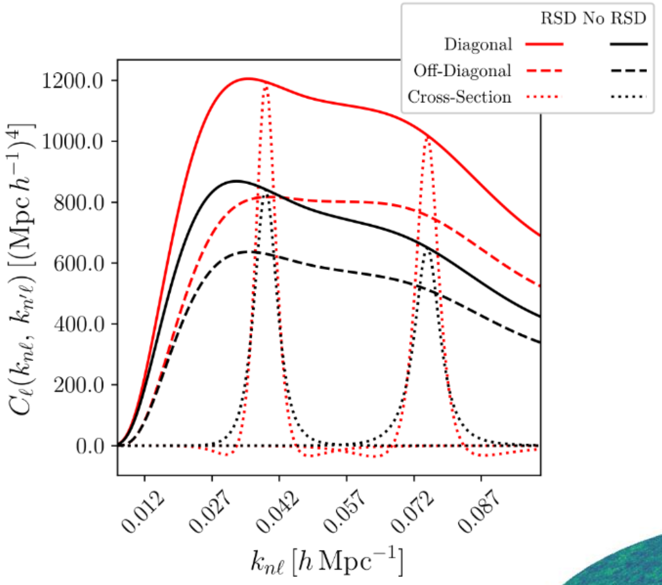
(Special thanks to Katarina Markovič)

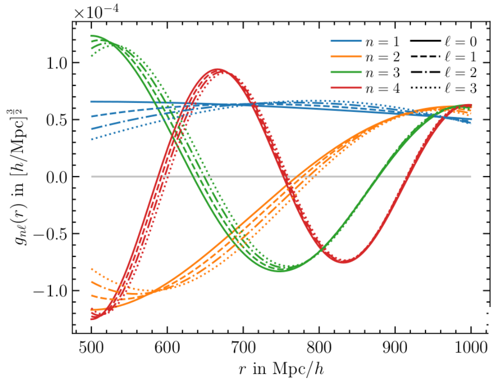

Roman-like
SPHEREx-like
Euclid-like
Input $P(k)$
Theory C
Measured $\bar{C}^A$
$\bar{C}_\ell(k,k)$ in $[h/\mathrm{Mpc}]^3$
k in h/Mpc
$\frac{1}{\bar{n}}$
Theory N^{shot}
Measured $\bar{N}^{\mathrm{shot}}$
$\bar{N}_\ell(k,k)$ in $[h/\mathrm{Mpc}]^3$
Henry S. Grasshorn Gebhardt, JPL/Caltech

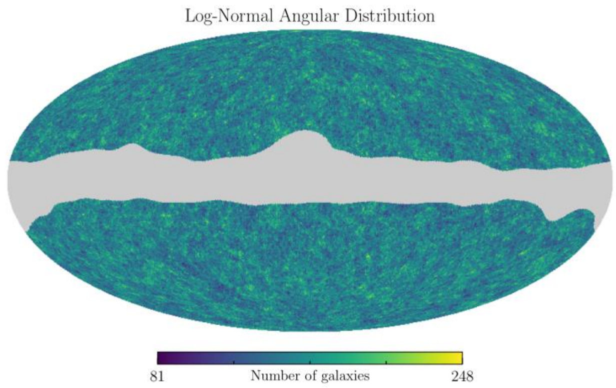
Summary & Conclusion

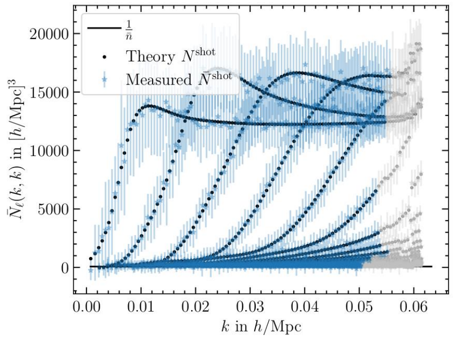
- SFB is theoretically optimal for **deep** and **wide** surveys
- Boundary conditions at r_{min} and r_{max} make SFB analysis more **numerically stable**
- SuperFaB is feasible for large surveys
- Future:
    - Apply to data!

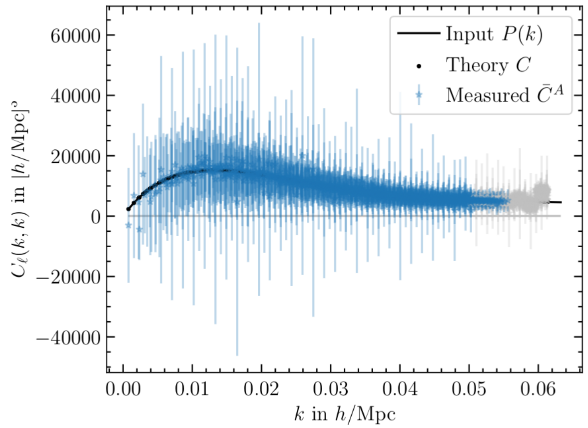
Paper: [arXiv:2102.10079]()
Julia code: [SphericalFourierBesselDecompositions.jl]()











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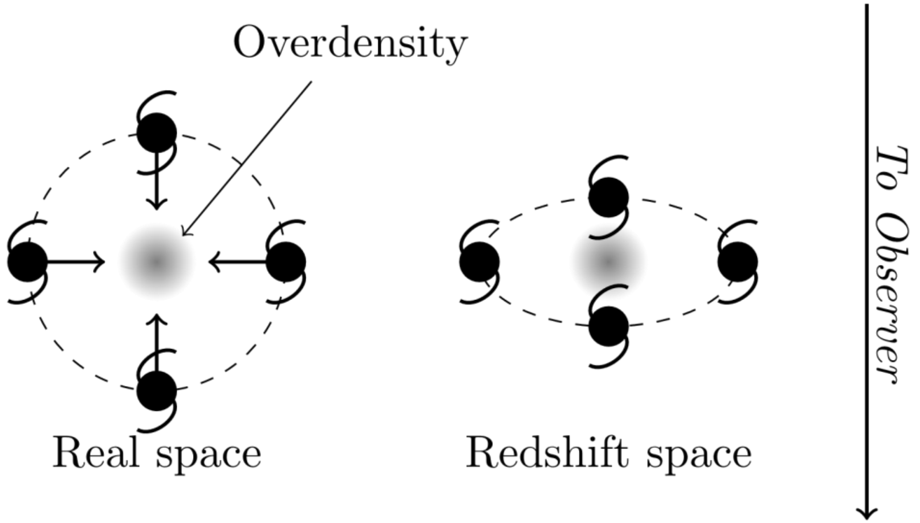
Redshift-space distortions I: Kaiser effect



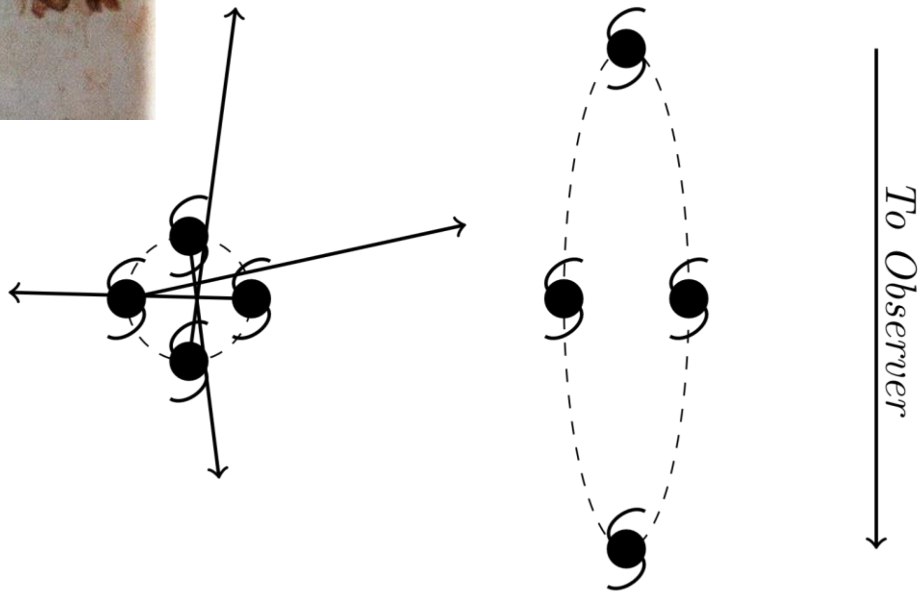
Coherent flow of galaxies towards overdensities leads to increased clustering along the line of sight direction.

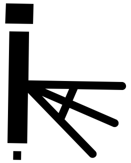
→ Non-zero quadrupole!

$$s = r + \frac{v_{\parallel}}{aH}$$

“Pancakes of God”

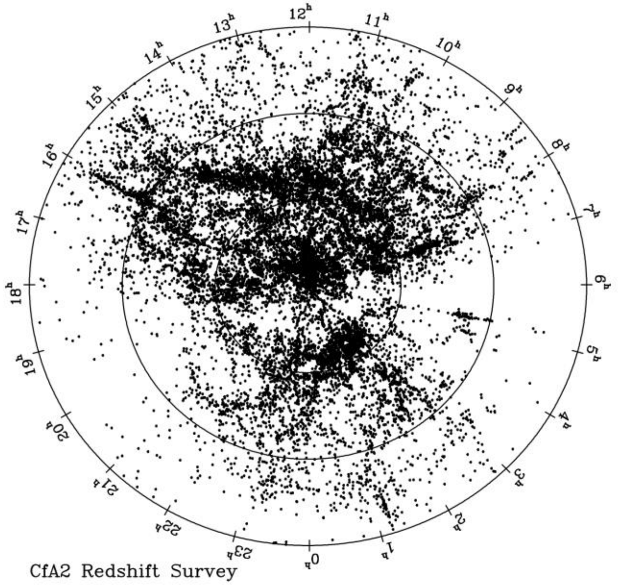
Redshift-space distortions II: Fingers of God



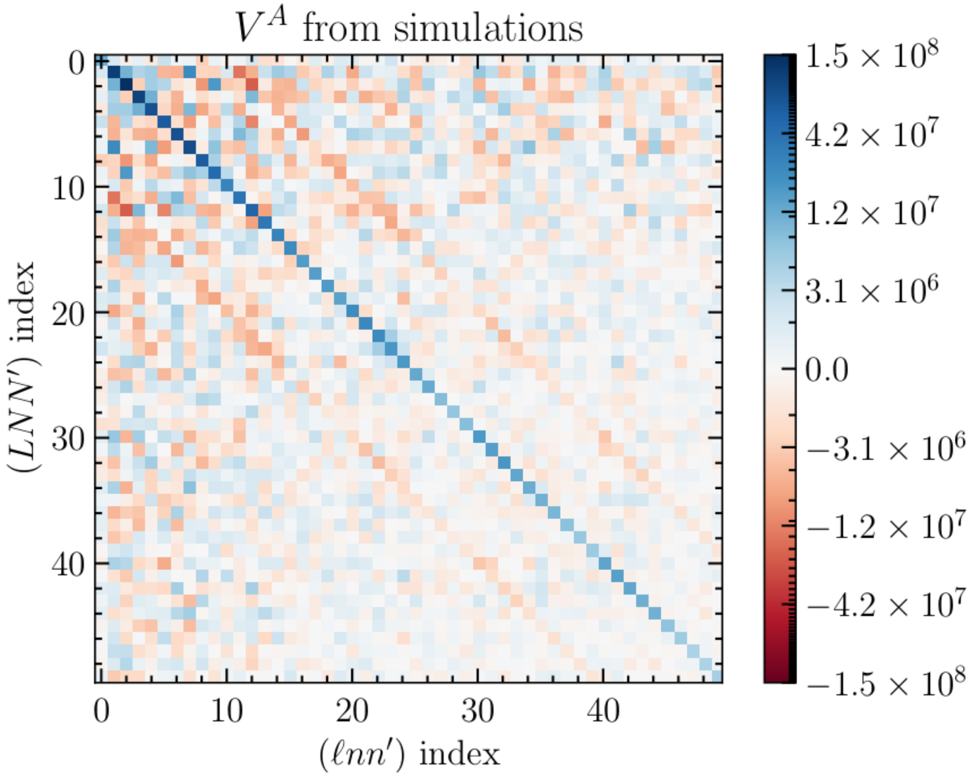


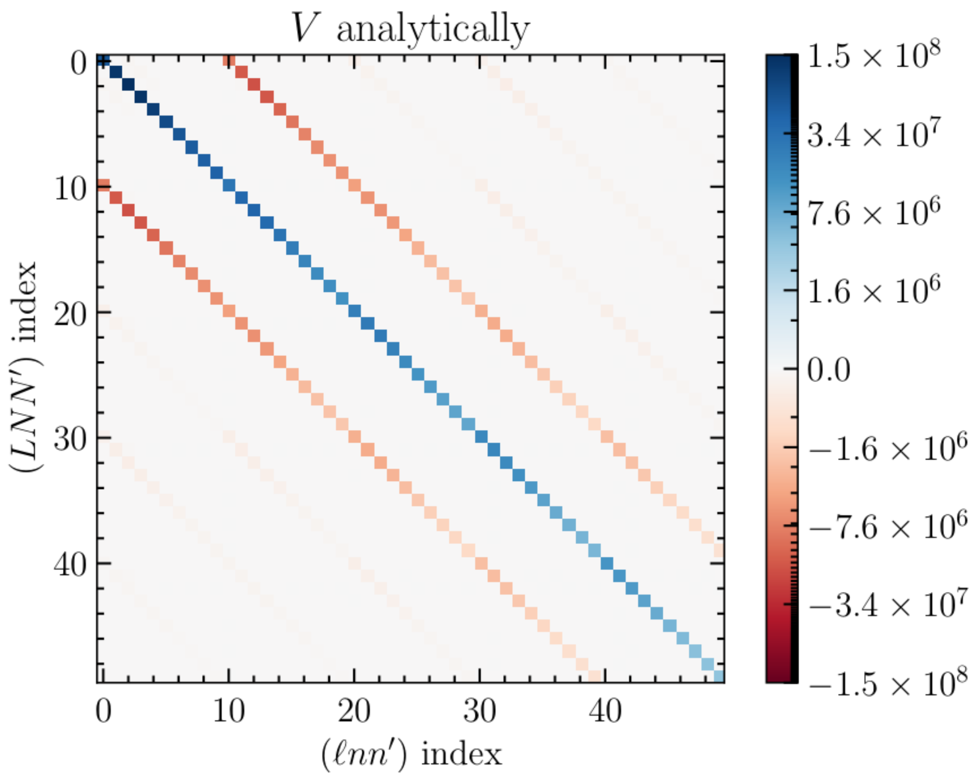
Stochastic motions of galaxies lead to a suppression of clustering along line-of-sight direction.

→ Small-scale suppression of power.

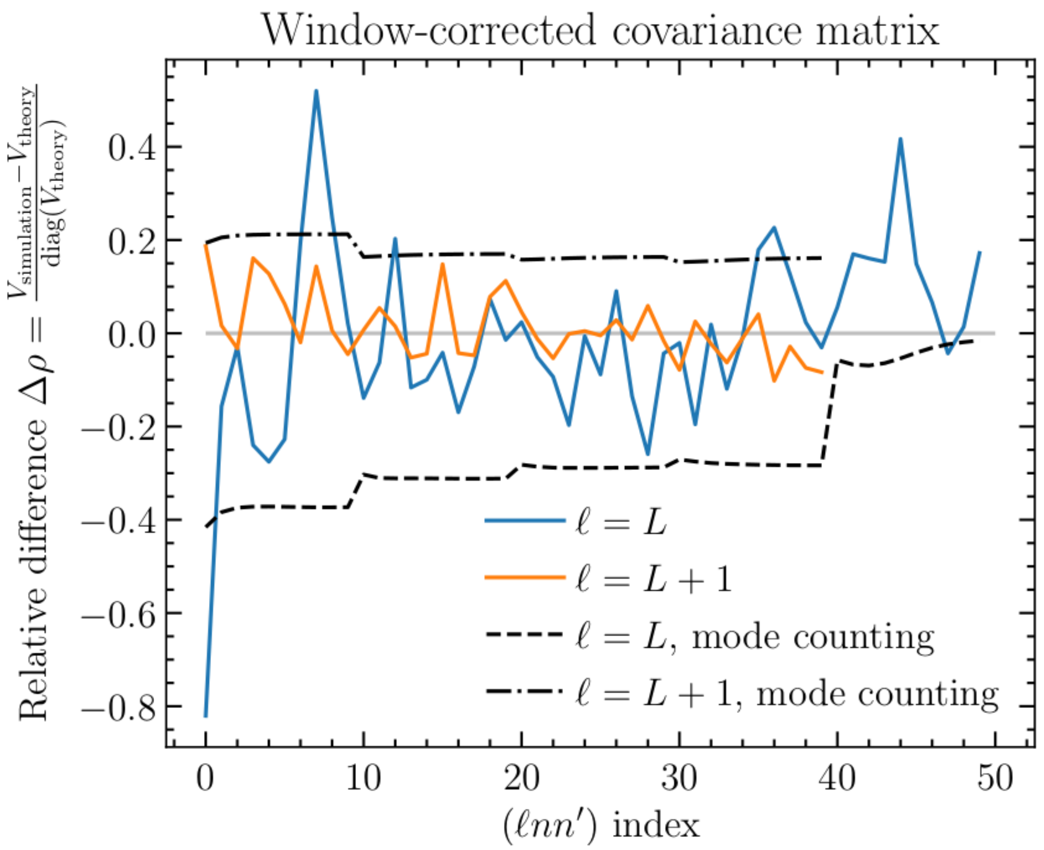


Covariance matrix V





~30% accurate covariance is fast



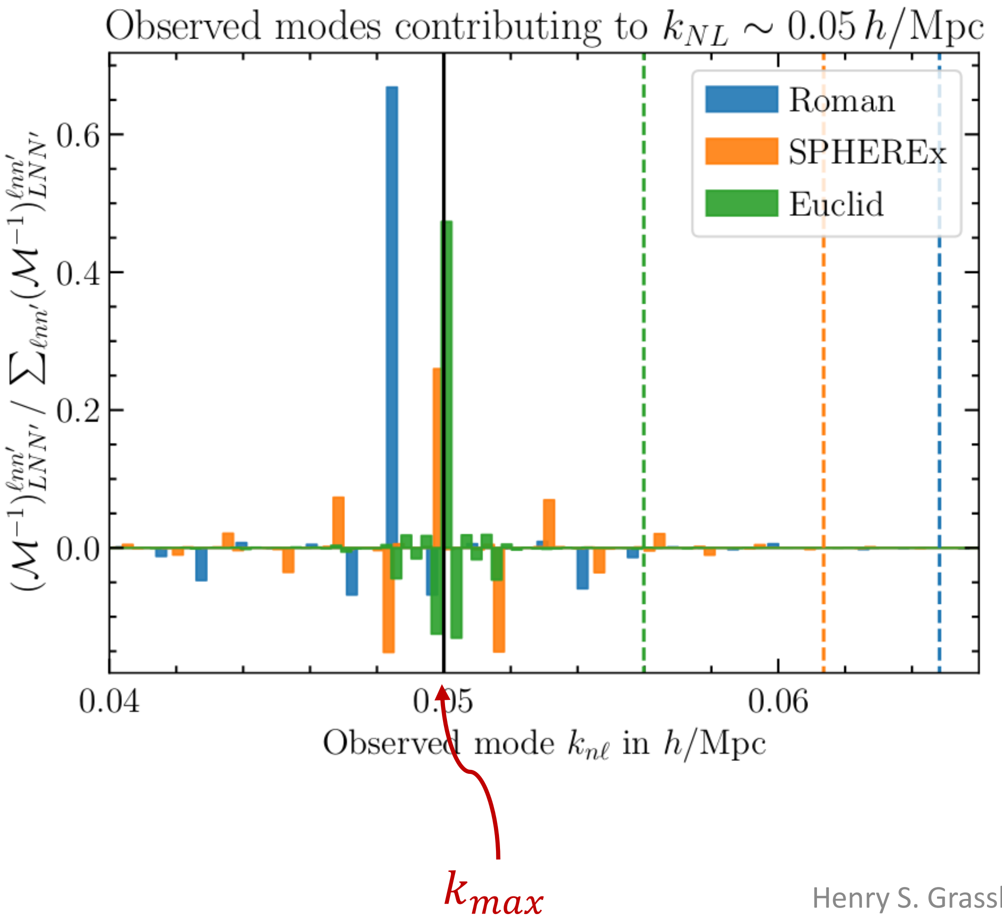
Mode-counting works OK:

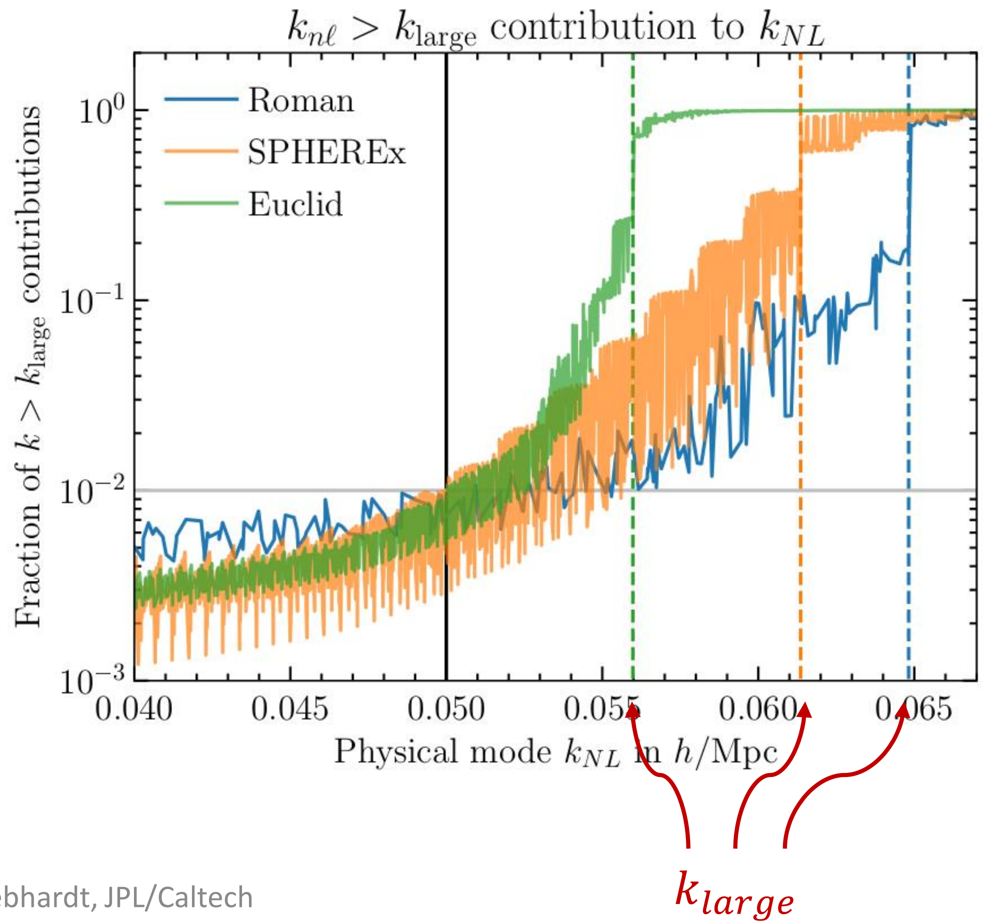
$$V_{LNN'}^{\ell n n'} \cong \frac{\delta_{\ell L}^{K}}{N_{modes}} \left[C_{\ell n N} C_{L n' N'} + C_{\ell n N'} C_{L n' N}\right]$$

$$N_{modes} = f_{vol}\,(2\ell + 1)\,\Delta\ell\,\Delta n$$

Full window deconvolution requires modes above k_{max}

$$\hat{C}^{\rm obs}_{\ell n n'} = \sum_{LNN'} \mathcal{M}^{LNN'}_{\ell n n'}\, \hat{C}^{A}_{LNN'}$$





Why a new code?

- We follow Samushia (2019) to introduce boundary conditions at both r_{min} and r_{max}

Spherical Bessels of 1. and 2. kind

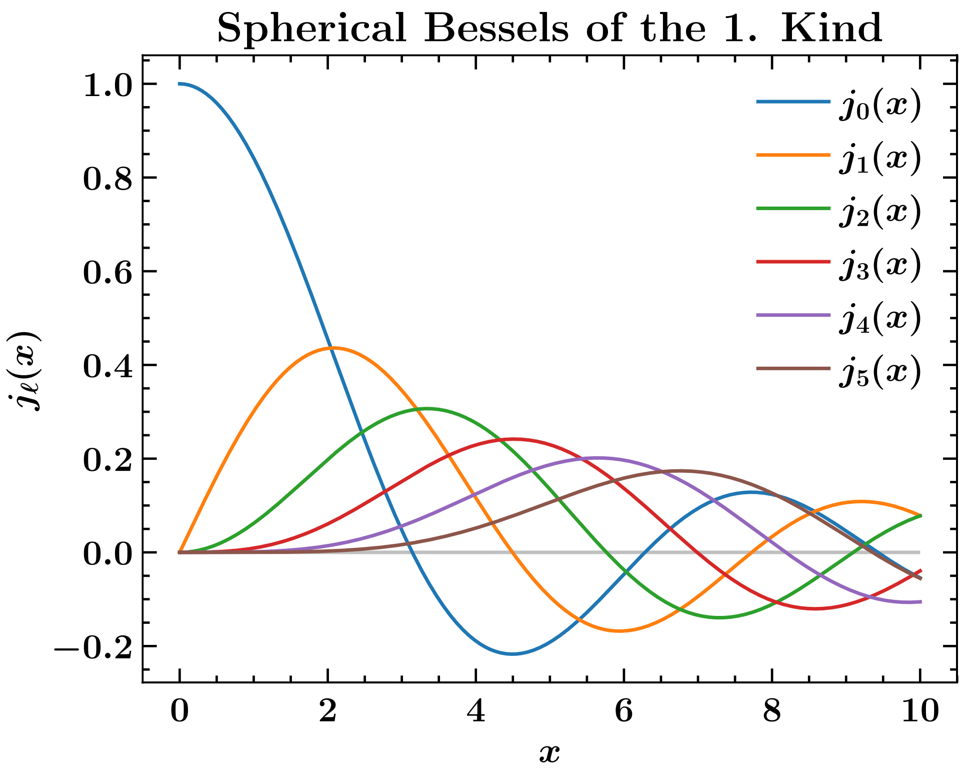
$$\nabla^2 f = -k^2 f$$

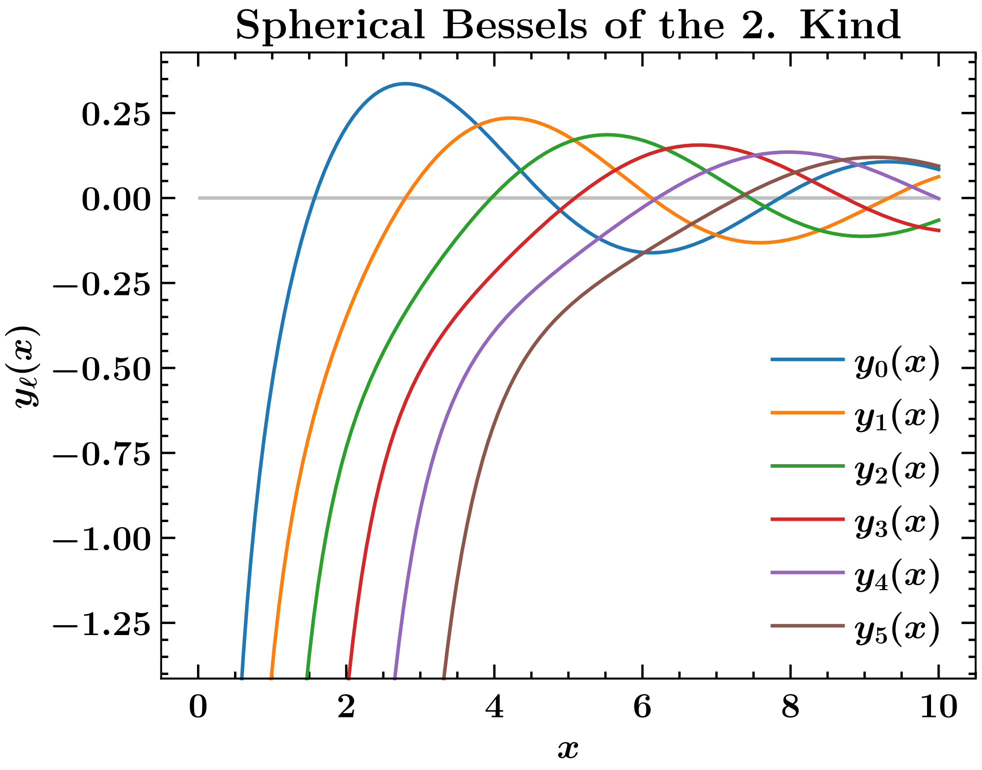
$$f_{\ell\mu}(k; r, \theta, \phi) = \left[c_j\, j_\ell(kr) + c_y\, y_\ell(kr)\right] \times \left[c_p\, P_\ell^\mu(\cos\theta) + c_q\, Q_\ell^\mu(\cos\theta)\right] \times \left[c_+\, e^{i\mu\phi} + c_-\, e^{-i\mu\phi}\right],$$

Spherical harmonics $Y_{\ell m}$

- Numerically more stable
- We use potential boundary conditions

Spherical Bessel and Neumann functions





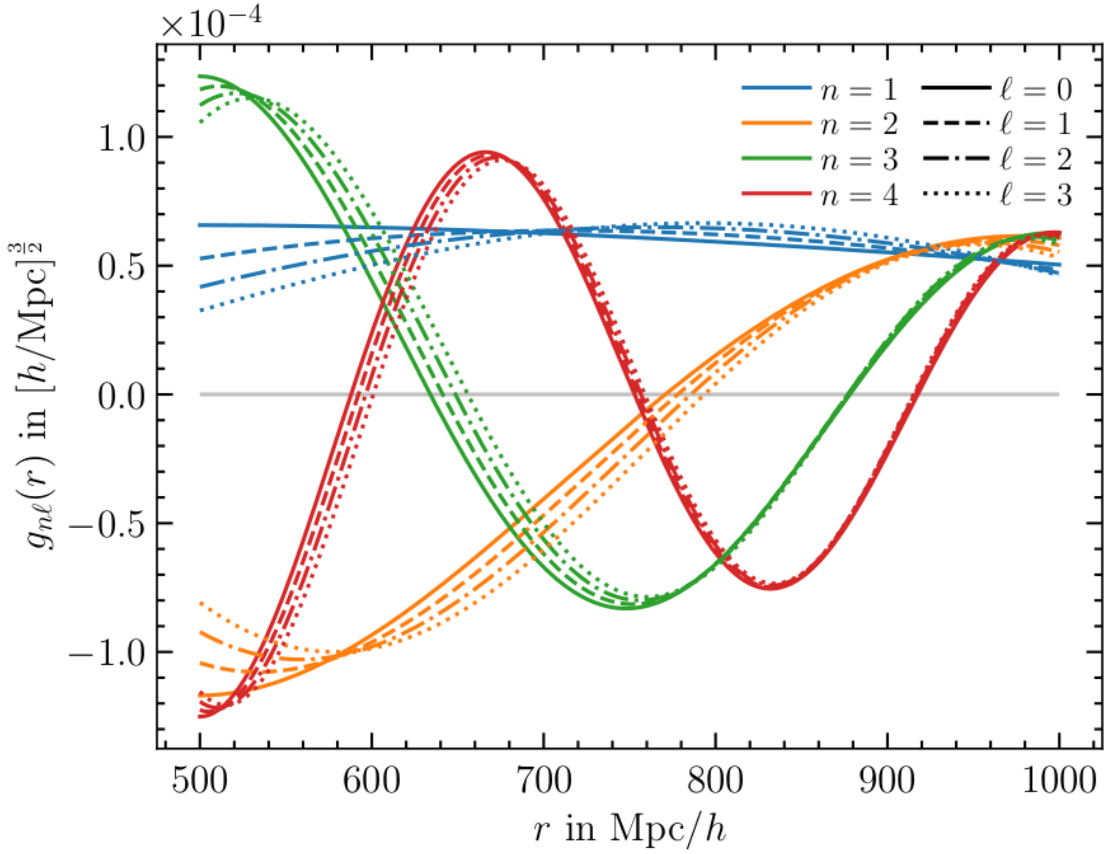
$$f_{\ell\mu}(k;r,\theta,\phi) = \left[c_j\, j_\ell(kr) + c_y\, y_\ell(kr)\right] \times \left[c_p\, P_\ell^\mu(\cos\theta) + c_q\, Q_\ell^\mu(\cos\theta)\right] \times \left[c_+\, e^{i\mu\phi} + c_-\, e^{-i\mu\phi}\right],$$

Radial basis functions

- Boundary conditions at both r_{min} and r_{max}
- Spherical Bessels of the second kind

$$g_{n\ell}(r) = c_{n\ell}\, j_\ell(k_{n\ell}\, r) + d_{n\ell}\, y_\ell(k_{n\ell}\, r)$$

➢Orthogonal basis where the survey is sensitive (Samushia 2019)



Limber's Approximation

Power spectrum

Fingers of God

Redshift evolution

$$C_\ell(k,k') = P(k)\, e^{-\sigma_u^2 k^2}\, \delta^D(k-k') \times \phi^2\left(\frac{\ell+\frac{1}{2}}{k}\right) D^2\left(\frac{\ell+\frac{1}{2}}{k}\right) b^2\left(\frac{\ell+\frac{1}{2}}{k}, k\right) \times \left[1 - \beta\left(f^\ell_{-2} + f^\ell_0 + f^\ell_2\right)\right]^2 .$$

SFB power spectrum

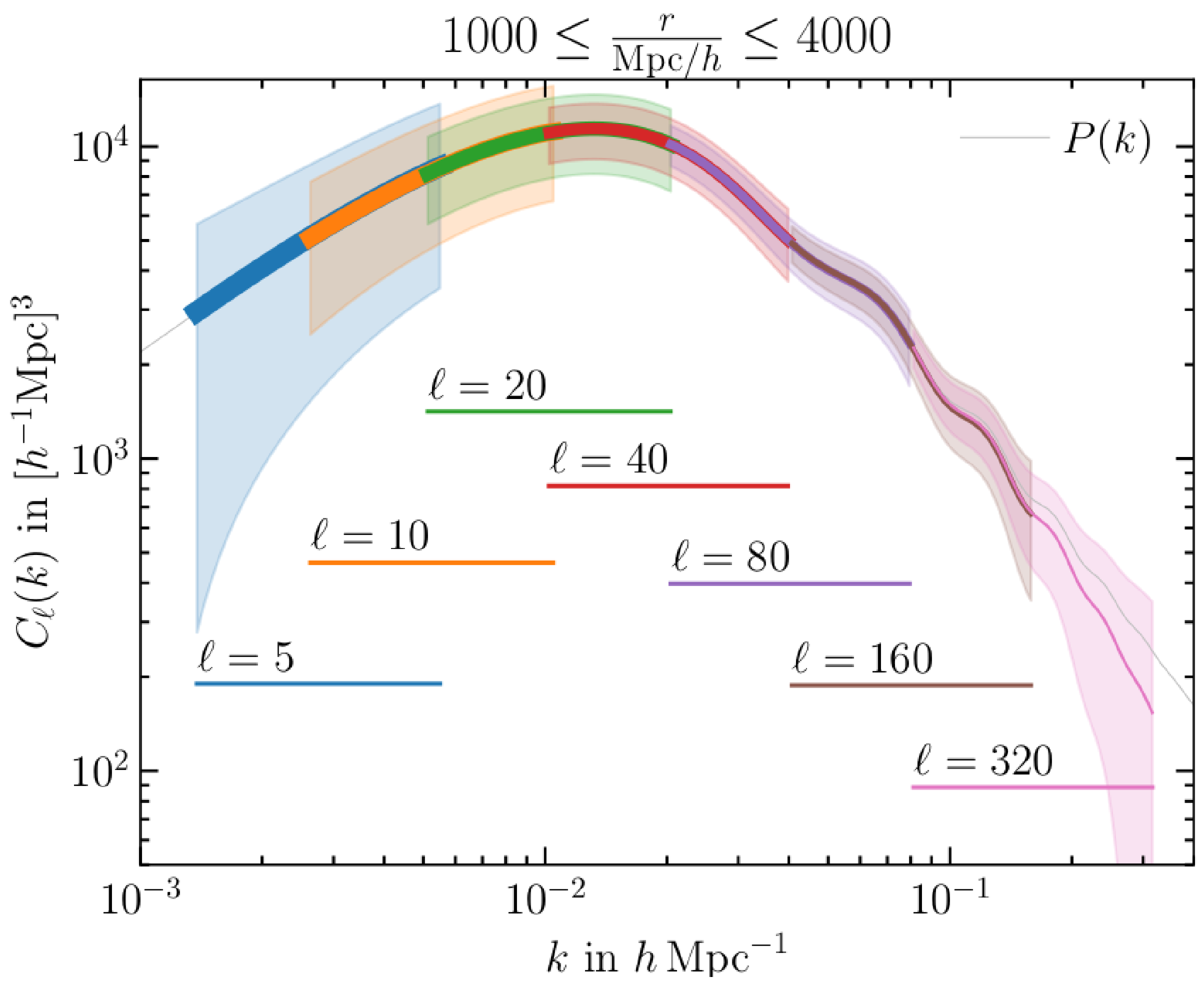
Scale-dependent bias

Linear Kaiser effect

The SFB power spectrum $C_\ell(k, k')$

- $b(z) \propto \frac{1}{D(z)}$

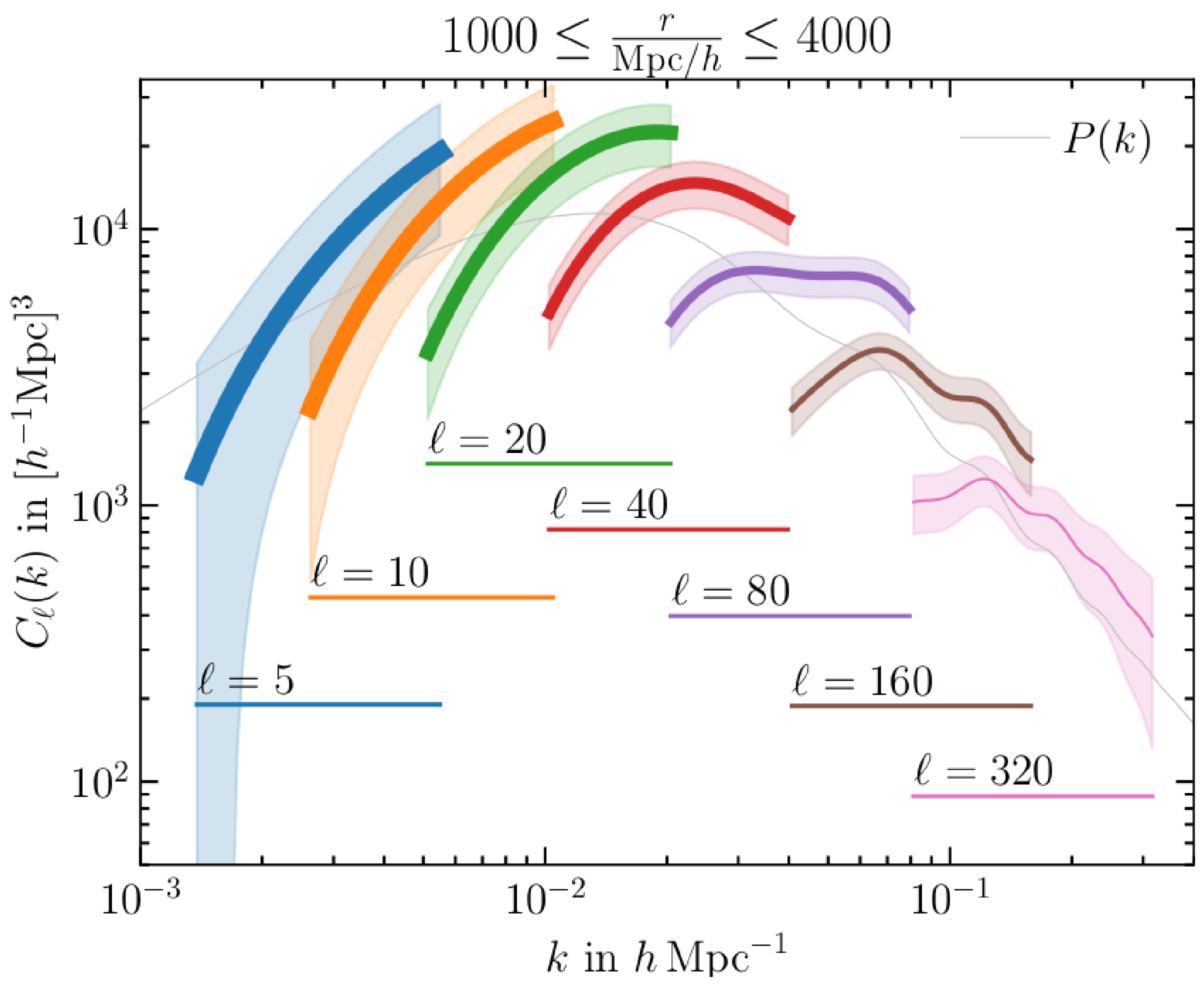
- Limber distance:
$$r = \frac{\ell + \frac{1}{2}}{k}$$

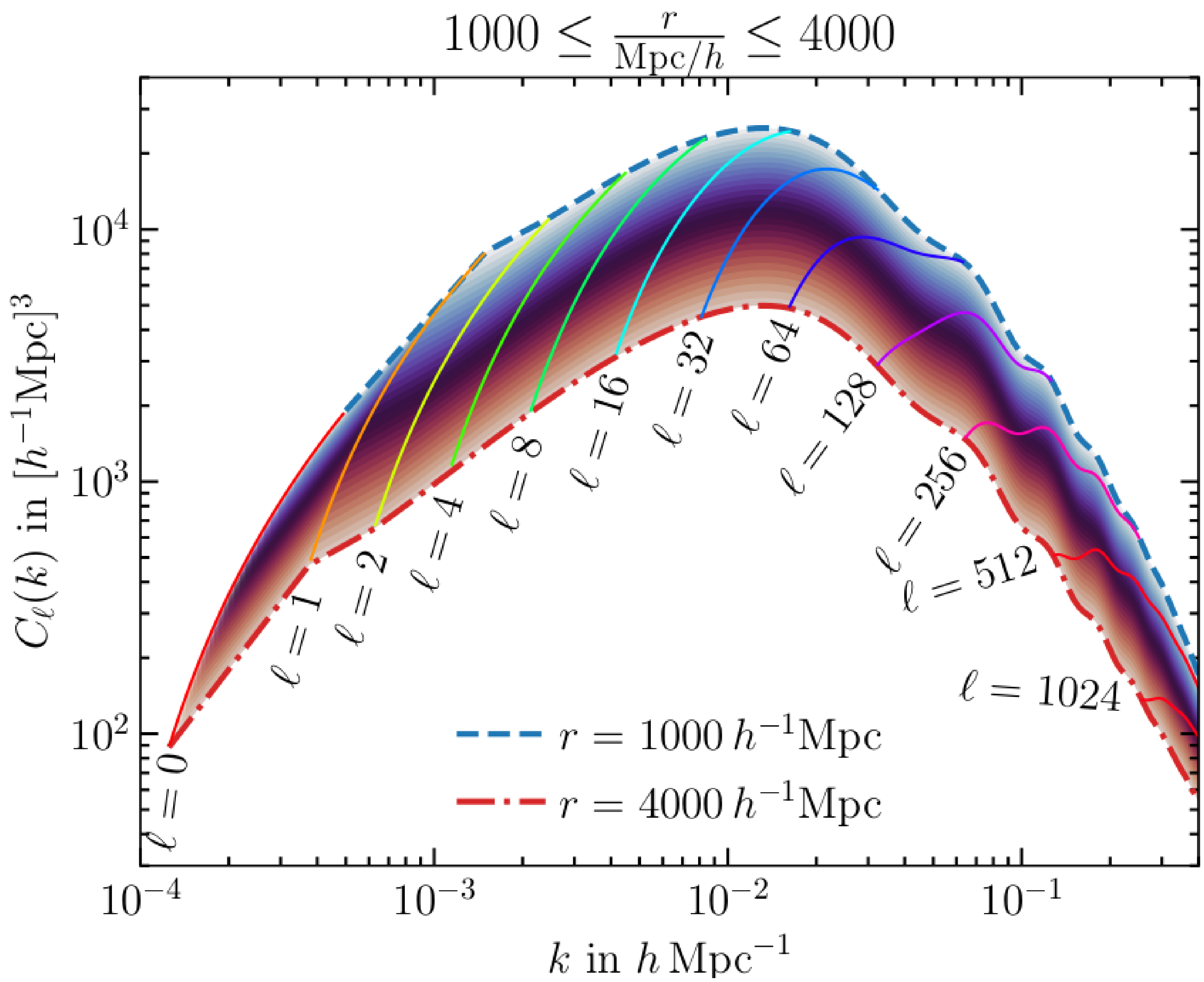


The SFB power spectrum $C_\ell(k, k')$

- $b(z) = const$
- Limber distance:

$$r = \frac{\ell + \frac{1}{2}}{k}$$





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